محاضرة رقم ٧ Math 3

Fuzzy

A Fuzzy set ~~A⊆R~~ $A \subseteq R^n$ (A_α α-cut set)

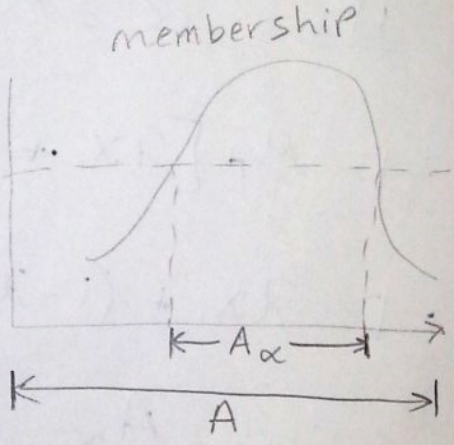
is Convex if and only if :-

$$\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \min\left[\mu_A(x_1), \mu_A(x_2)\right]$$

for all $x_1, x_2 \in \mathbb{R}^n$

الحل

Convex $\Longleftrightarrow$ الشرط يتحقق



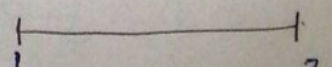
$\Rightarrow$ Let A_α is Convex for all.

$\Rightarrow \lambda x_1 + (1-\lambda)x_2 \in A_\alpha \quad 0 \leq \lambda \leq 1$

$\Rightarrow \mu_A(x_1) \geq \alpha \; ; \mu_A(x_2) \geq \alpha$

$\therefore \mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \alpha$

$1(\lambda) + 2(1-\lambda)$



$0 \leq \lambda \leq 1$

$$\Rightarrow \mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \alpha = \min\left(\mu_A(x_1), \mu_A(x_2)\right)$$

if $\mu_A(x_1) \leq \mu_A(x_2)$

الإتجاه العكس

Let $\mu_A[\lambda x_1 + (1-\lambda)x_2] \geq \min[\mu_A(x_1), \mu_A(x_2)]$

$A_\alpha \neq \phi$

if $\alpha = \mu_A(x_1) \leq \mu_A(x_2)$

$\Rightarrow \mu_A[\lambda x_1 + (1-\lambda)x_2] \geq \min[\alpha, \mu_A(x_2)]$

$\Rightarrow \lambda x_1 + (1-\lambda)x_2 \in A_\alpha$

$\therefore A_\alpha$ is Convex

Magnitude of fuzzy set

1 Scaler Cardinality :-

← يمثل هذا المقياس مجموع درجات الإنتماء لجميع عناصر الفئة.

$$|A| = \sum_{x \in A} \mu_A(x)$$

2 Relative Cardinality:-

$$||A|| = \frac{|A|}{|X|} = \frac{\sum_{x \in A} \mu_A(x)}{\text{no. of element of } x}$$

← هذا المقياس يمثل مقدار التأكد من صحة الـ (data) بالنسبة للعينة التي تحمل الفئة فئة فارغة.

EX: Consider the fuzzy set: short, medium, tall

Cm	short	middle	tall
14	1	0	0
15	1	0	0
16	0.9	0.1	0
17	0.7	1	0
18	0.3	0.8	0.3
19	0	0	1

1. Compare the support of each set
2. Compare the α-cut of each set at $\alpha = 0.5$
3. $|short|$ and $||short||$

Solution

1

$$\text{supp}(A) = \{x : \mu(x) > 0\}$$

$$\text{supp}(\text{short}) = \{14, 15, 16, 17, 18\}$$

$$\text{supp}(\text{medium}) = \{16, 17, 18\}$$

$$\text{supp}(\text{tall}) = \{18, 19\}$$

2 $$A_\alpha = \{x : \mu(x) \geq \alpha\}$$

$$(\text{short})_{0.5} = \{14, 15, 16, 17\}$$

$$(\text{medium})_{0.5} = \{17, 18\}$$

$$(\text{tall})_{0.5} = \{19\}$$

3

$$|\text{short}| = 1 + 1 + 0.9 + 0.7 + 0.3 = 3.9$$

$$\|\text{short}\| = \frac{3.9}{6}$$

4 Lec 7

→ operation on Fuzzy set:-

① Complement:

$$\mu_{\bar{A}}(x) = -\mu_A(x) \; : \; x \in X$$

② union :-

$$\mu_{A \cup B}(x) = \max\{\mu_A(x), \mu_B(x)\}$$

③ intersection :-

$$\mu_{A \cap B}(x) = \min\{\mu_A(x), \mu_B(x)\}$$

EX Let $\tilde{A} = \frac{0.3}{1} + \frac{0}{2} + \frac{0.4}{3} + \frac{0.8}{4} + \frac{1}{5}$,

$\tilde{B} = \frac{0.2}{1} + \frac{0.3}{2} + \frac{0.1}{3} + \frac{0.2}{4} + \frac{0.4}{5}$ Find

① $\bar{\tilde{A}}$

② $\tilde{A} \cup \tilde{B}$

③ $\tilde{A} \cap \tilde{B}$

④ $\tilde{A} - \tilde{B}$

⑤ $\tilde{A} \Delta \tilde{B}$

Solution

① $\overline{\tilde{A}} = \frac{0.7}{1} + \frac{1}{2} + \frac{0.6}{3} + \frac{0.2}{4} + \frac{0}{5}$

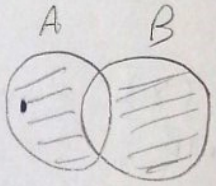
② $\tilde{A} \cup \tilde{B} = \frac{0.3}{1} + \frac{0.3}{2} + \frac{0.4}{3} + \frac{0.8}{4} + \frac{1}{5}$

③ $\tilde{A} \cap \tilde{B} = \frac{0.2}{1} + \frac{0}{2} + \frac{0.1}{3} + \frac{0.2}{4} + \frac{0.4}{5}$

④ $\tilde{A} - \tilde{B} = \tilde{A} \cap \overline{\tilde{B}} = \frac{0.2}{1} + \frac{0}{2} + \frac{0.1}{3} + \frac{0.2}{4} + \frac{0.4}{5}$

⑤ $\tilde{A} \Delta \tilde{B} = (\tilde{A} \cap \overline{\tilde{B}}) \cup (\tilde{B} \cap \overline{\tilde{A}})$.

~~$\tilde{A} \cap \tilde{B} = \frac{0.2}{1} + \frac{0}{2} + \frac{0.1}{3} + \frac{0.2}{4} + \frac{0.4}{5}$~~



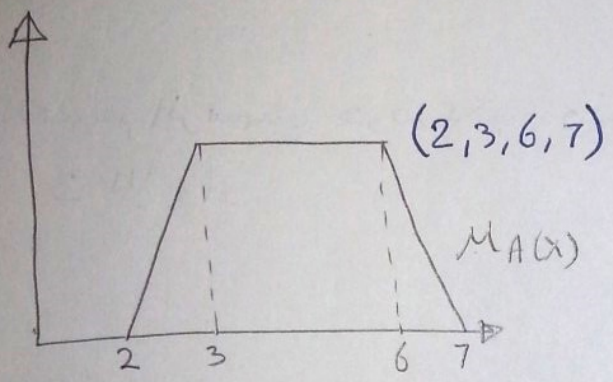
$A \Delta B = (A - B) \cup (B - A)$

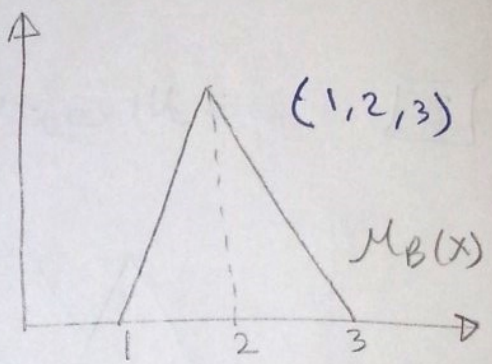
$\tilde{B} \cap \overline{\tilde{A}} =$

كذلك

← في الحالة المتصلة عندما تكون $\mu(x)$ تأخذ قيم
في الحالة المنفصلة (discrete)

EX Graphically represent the fuzzy set operation if the member ship.



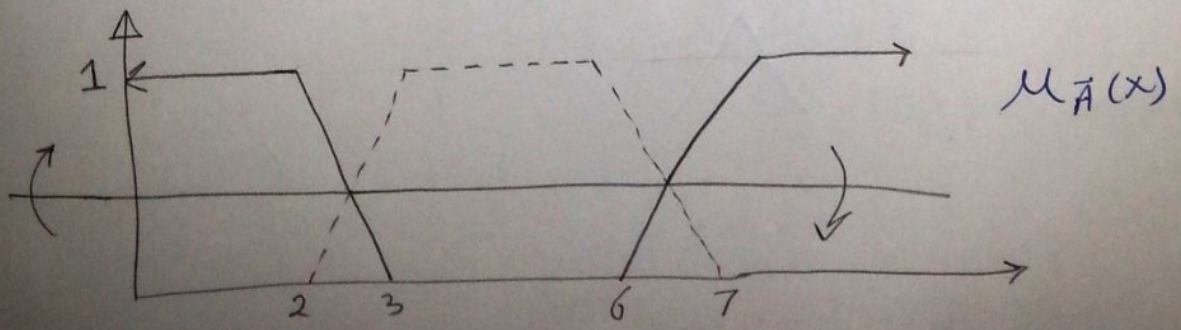


Find

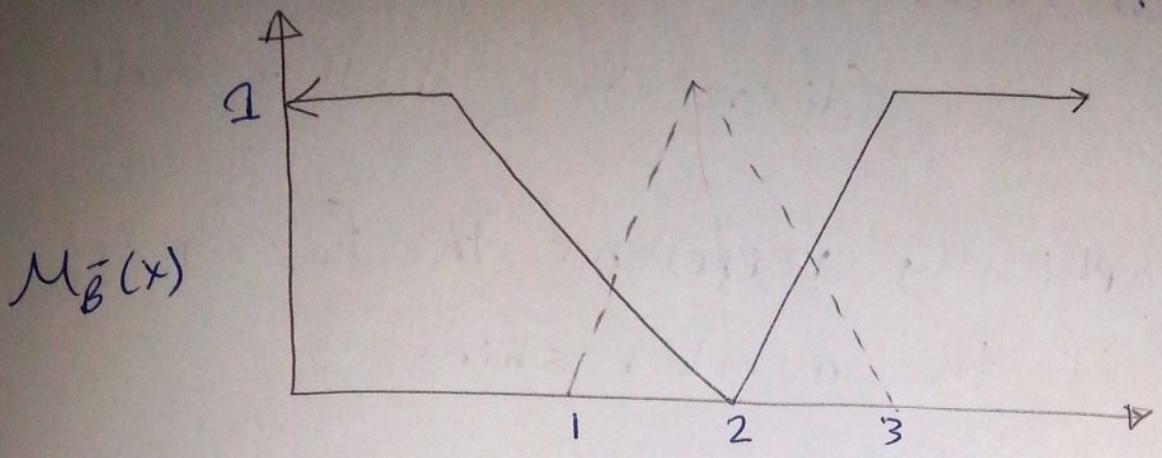
1. $\mu_{\bar{A}}(x)$
2. $\mu_{\bar{B}}(x)$
3. $\mu_{A \cup B}(x)$
4. $\mu_{A \cap B}(x)$

Sol

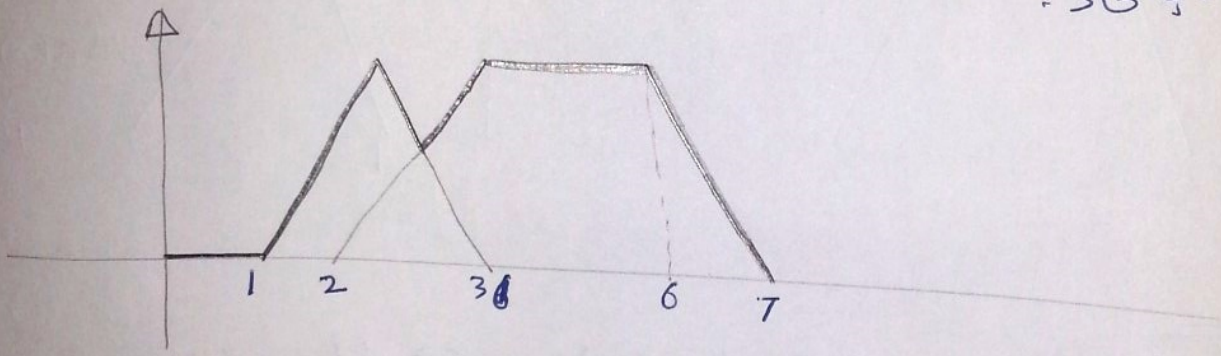
1



2



3 ← هنرسم الرسمتين فوق بعض ونمشي على الحرف اللي فوق في الإتحاد.



4 في التقاطع هنرسم الرسمتين على بعض ونمشي على الحرف اللي تحت.